AD-A133 291 DYNAMICS OF A HIGH CURRENT ELECTRON RING IN A CONVENTIONAL BETATRON ACCELERATOR(U) NAVAL RESEARCH LAB WASHINGTON DC C A KAPETANAKOS ET AL. 14 SEP 83 1/1

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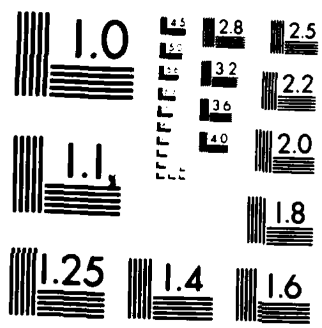

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NRL Memorandum Report 5108	2. GOVT ACCESSION NO. AD-A133 291	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) DYNAMICS OF A HIGH CURRENT ELECTRON RING IN A CONVENTIONAL BETATRON ACCELERATOR		5. TYPE OF REPORT & PERIOD COVERED Interim report on a continuing NRL problem. 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) C.A.Kapetanakos, S.J. Marsh,* and P. Sprangle		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Research Laboratory Washington, DC 20375		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 61153N; RR011-09-41; 47-0924-0-3
11. CONTROLLING OFFICE NAME AND ADDRESS Office of Naval Research Arlington, VA 22217		12. REPORT DATE September 14, 1983 13. NUMBER OF PAGES 40
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

*Present address: Sachs/Freeman Associates, Bowie, MD 20715

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Betatron
High current electron beams
Accelerators

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Analytical and computer simulation results are reported on the dynamics of a high current electron ring in a conventional betatron accelerator. These studies include, in addition to the external and self fields, the effects of surrounding walls and toroidal corrections. Our results show that, in contrast with the modified betatron, the equilibrium in a conventional betatron is incompatible with either large thermal energy spread or emittance. However, as in the modified betatron, the energy mismatch and the diffusion of the self magnetic field impose stringent

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20. ABSTRACT (Continued)

constraints on the accelerator. Finally, in both devices, it is necessary to keep the ratio of the ring to the torus minor radii much less than unity.

CONTENTS

DYNAMICS OF A HIGH CURRENT ELECTRON RING IN A CONVENTIONAL BETATRON ACCELERATOR

I. Introduction

Over the last few years, there is increasing interest on the development of ultra-high current accelerators. Currently, several laboratories[1-17] are engaged in studies that are aimed to assess the feasibility of developing such accelerators.

Induction acceleration, either in linear[1,2] or cyclic geometries,[3-17] is presently the most popular approach among the various accelerating schemes. Although the beam dynamics is relative simple in linear devices, their long length and high cost make them unattractive, when high energies are desired. For this reason, progressively more attention is focused on cyclic induction accelerators.

So far three different cyclic induction accelerators have been proposed: the conventional betatron,[18,19] the modified betatron[4-16] and the stellatron.[17] The modified betatron includes in addition to the time varying betatron magnetic field that is responsible for the acceleration, a strong toroidal magnetic field that substantially improves the stability of the conventional betatron. In the stellatron, the addition of a stellarator field to the modified betatron substantially reduces the displacement of the orbit that is due to energy mismatch.

The dynamics of a high current electron ring confined in a modified betatron configuration has been studied extensively over the last two years.[12-15] As a result of the finite ν/γ of the electron ring a host of new phenomena either surfaced or became more pronounced. In this paper, we analyze and discuss the dynamics of a finite ν/γ electron ring confined in a

Manuscript approved May 12, 1983.

conventional betatron. The present work includes both analytical and computational studies for "cold" and "hot" electron rings. The results indicate that, in contrast with the modified betatron, the equilibrium in a conventional betatron is incompatible with either large thermal energy spread of emittance. In addition, it was found that for a "hot" ring, i.e., a ring with toroidal thermal energy spread, the radial ($\hat{\epsilon}_r$) component of the rms emittance oscillates in time, while the vertical component ($\hat{\epsilon}_z$) remains constant. Finally, the energy mismatch and the diffusion of the self magnetic field, as in the modified betatron, impose stringent constraints on the accelerator.

II Transverse Dynamics

a. Macroscopic (beam) motion without toroidal corrections

In this sub-section we study the dynamics of a high current electron beam, including the effect of surrounding conducting walls. However, toroidal correction associated with the fields are neglected. These corrections are considered in sub-section c.

Consider a pencil-like electron beam inside a straight, perfectly conducting cylindrical pipe of circular cross-section as shown in Fig. 1. The center of the beam is located at a distance Δr, Δz from the center of the minor cross-section of the pipe. As a result of the induced charges on the wall, the center of the beam will experience a radial, outward directed force, which for small displacements, i.e., Δr, $\Delta z \ll a$ is given by

$$\vec{F}_E = 2\pi e^2 n_o (r_b/a)^2 \{\Delta r \hat{e}_r + \Delta z \hat{e}_z\}, \tag{1}$$

where a is the cylinder radius and n_o the uniform beam density.

Similarly, as a result of the induced current on the wall, the center of the beam will experience a radial force that is directed toward the opposite direction than F_E and is given by

$$\vec{F}_B = - \beta_o^2 \vec{F}_E. \tag{2}$$

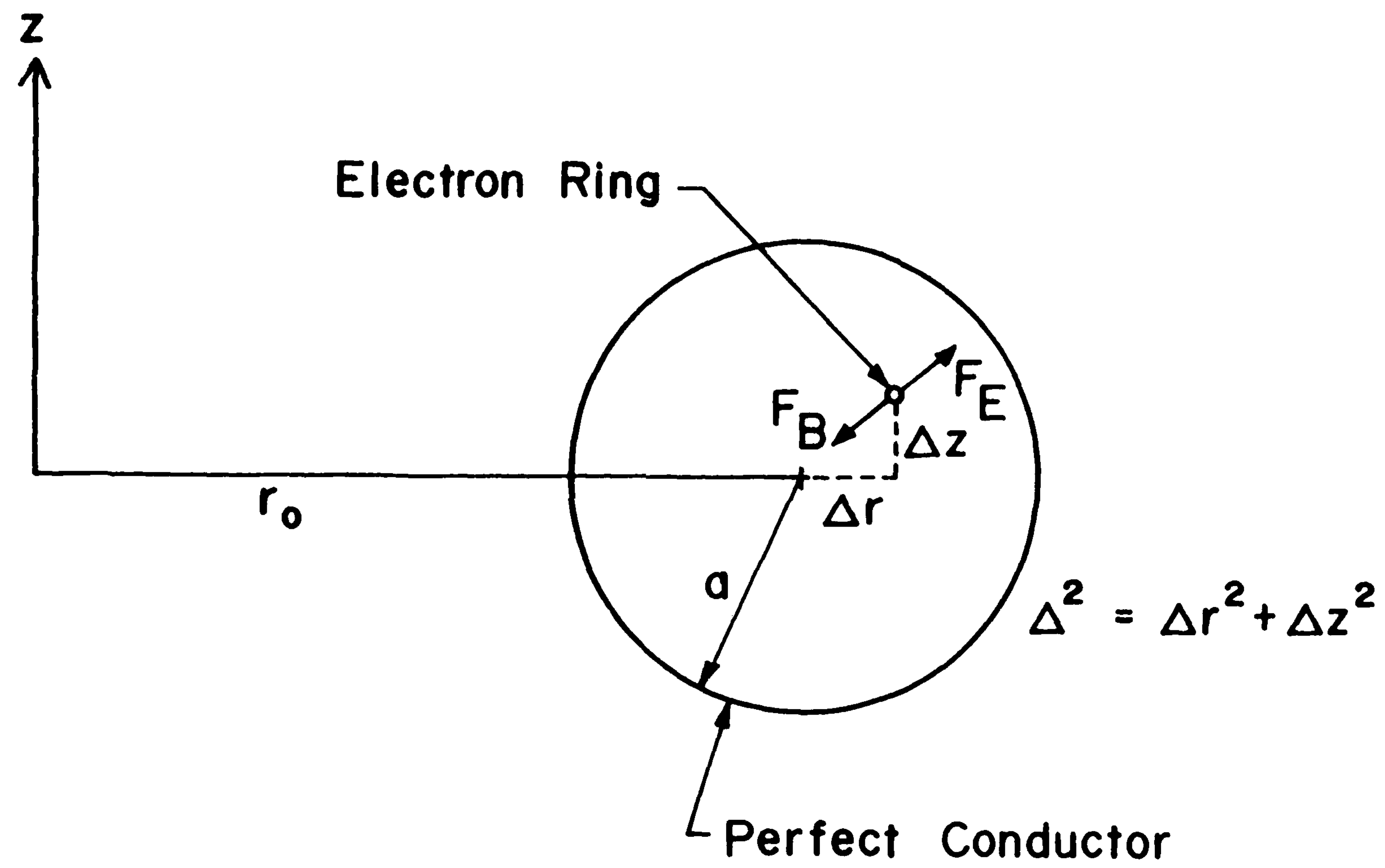


Fig. 1. Wall (image) forces acting on a pencil-like beam, situated inside a perfectly conducting cylindrical pipe.

In addition to the wall forces, the ring experiences the effect of the external fields, which are assumed to vary as

$$B_z(r,t) \cong B_{oz}(t)\ \{1 - n\ (r - r_o)/r_o\}, \tag{3a}$$

$$B_r(r,t) \cong - B_{oz}(t)\ nz/r_o, \tag{3b}$$

and

$$E_\theta\ (r,t) = - \frac{1}{rc} \int_o^r\ r'dr'\ \frac{\partial B_z}{\partial t}\ (r',\ t). \tag{3c}$$

In the above equations $B_z(r,t)$ is the axial and $B_r(r,t)$ is the radial component of the betatron field, $E_\theta(r,t)$ is the induced electric field and n is the external field index.

Using the induced fields of Eqs. (1) and (2), and the external field of Eqs. (3a) and (3b), the equations describing the temporal linear evolution of the beam's center, for time independent applied fields, are:

$$\ddot{\Delta r} + \tilde{\omega}_r^{\ 2}\ \Delta r = (\frac{\Omega_{oz}}{\gamma_o})\ \frac{\langle\delta P_\theta\rangle}{\gamma_o m r_o}, \tag{4a}$$

and

$$\ddot{\Delta z} + \tilde{\omega}_z^{\ 2}\ \Delta z = o, \tag{4b}$$

where $\tilde{\omega}_r^{\ 2} = (\Omega_{oz}/\gamma_o)^2(1 - n - n_s \frac{r_b^{\ 2}}{a^2})$, $\tilde{\omega}_z^{\ 2} = (\Omega_{oz}/\gamma_o)^2(n - n_s \frac{r_b^2}{a^2})$, (5)

$\Omega_{oz} = eB_{oz}/mc$, $\frac{\delta\gamma_o}{\gamma_o} = \frac{\beta_o\langle\delta P_\theta\rangle}{\gamma_o m r_o c}$ and δP_θ is the difference between the canonical

angular momentum of an electron at (r,z) and its corresponding value at the equilibrium orbit (r_o,o). The average is over initial coordinates and velocities. Equations (4) and (5) do not include the self electric and self magnetic fields, because both these fields are zero at the center of a straight beam.

In Eq. (4a), $\delta\gamma_o = \beta_o\langle\delta P_\theta\rangle/mr_oc$ indicates the energy mismatch, i.e., the difference between the energy of the reference electron (moving along the axis of the beam) and the energy required for the same electron to move on the equilibrium orbit (r_o,o). The solution of Eqs. (4), for time independent fields, is

$$\Delta r = \frac{\Omega_{oz}\langle\delta P_\theta\rangle}{\gamma_o^2\tilde{\omega}_r^2 r_o m} + \sum_j c_j\, e^{i\omega_j t} + \text{c.c}, \tag{6}$$

and

$$\Delta z = \sum_j c_j\, e^{i\omega_j t} + \text{c.c}, \tag{7}$$

where c_j are constants and

$$\omega_j^2 = \begin{cases} \tilde{\omega}_r^2 \\ \tilde{\omega}_z^2 \end{cases} . \tag{8}$$

The first term on the RHS of Eq. (6) gives the displacement of the center of the orbit from the center of the surrounding cylindrical pipe and can be

written as

$$\frac{\Delta r_o}{r_o} = \frac{\langle \delta P_\theta \rangle / \gamma_o}{(\Omega_{oz}/\gamma_o)\ (1 - n - n_s r_b{}^2/a^2) m r_o^2} = \frac{(\delta\gamma_o/\gamma_o)}{\beta_o^2\ (1-n-n_s\ r_b{}^2\ /a^2)}. \tag{9}$$

The displacement of the orbit's center because of the energy mismatch imposes very stringent constraints on the injector. This becomes apparent when we consider some limiting cases. For example, when $n={}^1/2$ and $n_s r_b{}^2/a^2 \ll 1$, Eq. (9) is reduced to

$$\frac{\Delta r_o}{r_o} \approx 2\ (\delta\gamma_o/\gamma_o). \tag{10}$$

Equation (10) predicts that for a major radius r_o = 100 cm, the ratio $\delta\gamma_o/\gamma_o$ should be less than 1% in order that the displacement of the orbit to be less than 2 cm. The condition $\delta\gamma_o/\gamma_o < 1\%$ requires that the uncertainty in energy should be less than 35 KeV, when the energy of the injected beam is 3 MeV.

For the initial conditions $\Delta r = \Delta r(o)$ and $\Delta z(o) = \Delta\dot{r}(o) = \Delta\dot{z}(o) = o$, Eqs. (6) and (7) give

$$\Delta r(t) = \Delta r_o + \left(\Delta r(o) - \Delta r_o\right) \cos\ (\bar{\omega}_r t), \tag{11a}$$

and

$$\Delta z(t) = o, \tag{11b}$$

i.e., the center of the beam oscillates along the radial direction around Δr_o.

Equations (11) are in good agreement with the results of computer simulation shown in Fig. 2. The values of the various parameters are listed in Table I. The center of the ring perform sinusoidal oscillations around the equilibrium position that is located 99 cm from the major axis ($\Delta r_o = -1$ cm) with a frequency $\tilde{\omega}_r$ given by Eq. (5), which gives a period of about 28 nsec, in agreement with the computer results. Such a pure radial motion cannot occur in a Modified Betatron configuration, because the toroidal magnetic field couples the r and z motions.

The orbit of the beam's center is not always a straight line. For example, when $\Delta r(o)$ and $\Delta\dot{z}(o) \neq o$ but $\Delta\dot{r}(o) = \Delta z(o) = o$

$$\Delta r = \Delta r_o + \left(\Delta r(o) - \Delta r_o\right) \cos \tilde{\omega}_r t,$$

and

$$\Delta z = \left(\frac{\Delta\dot{z}(o)}{\tilde{\omega}_z}\right) \sin \tilde{\omega}_z t,$$

which for $\tilde{\omega}_r = \tilde{\omega}_z$ gives an ellipse as shown in Fig. 3.

The linearized Eqs. (1) and (2) are based on the assumption that $\Delta r_{/a}$ and $\Delta z_{/a} \ll 1$. If this assumption is not satisfied, it is shown in the Appendix A that for an arbitrary minor radius beam of uniform charge and current density the fields at the center of the straight beam are given by

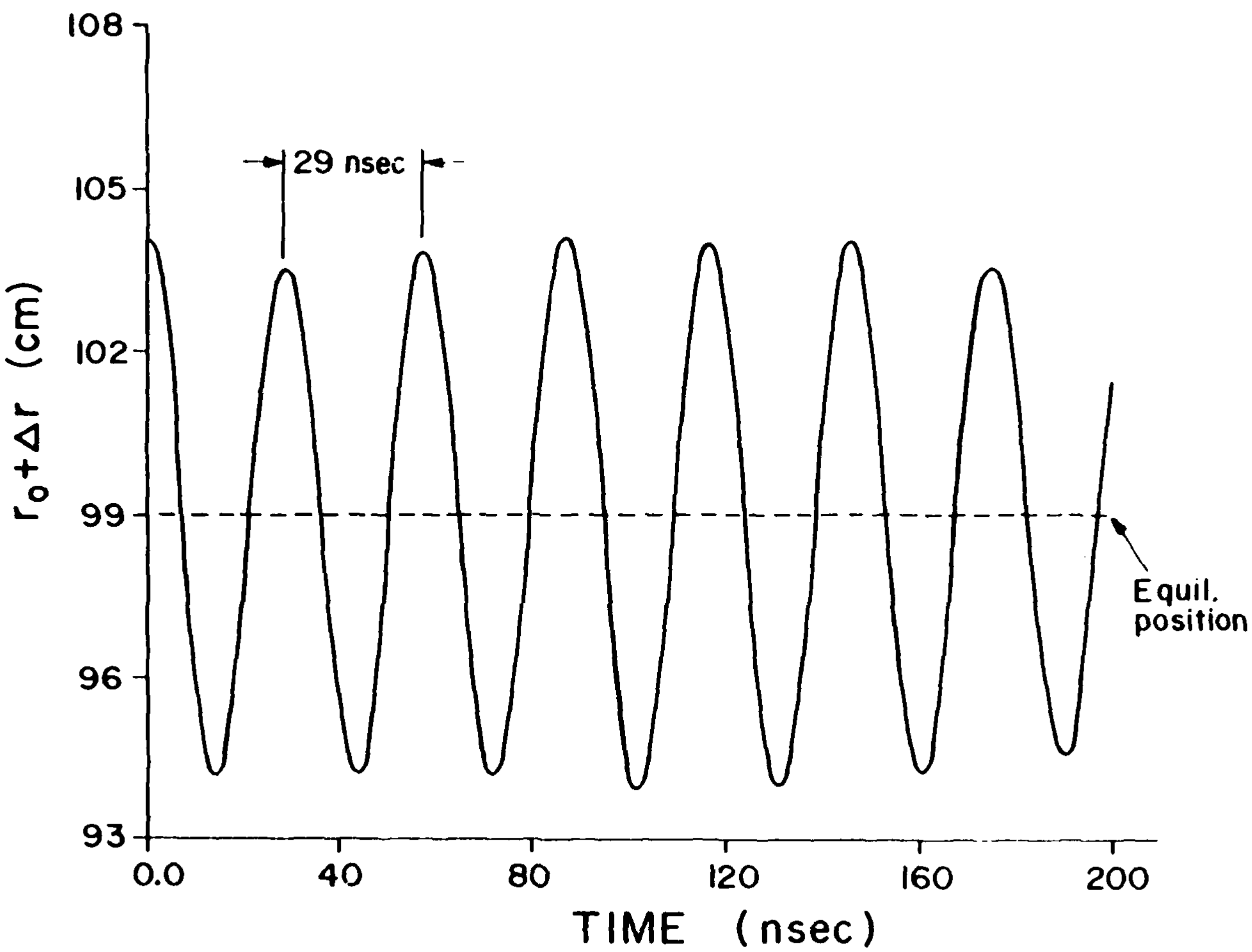


Fig. 2. Radial oscillations around the equilibrium position performed by the center of the electron ring. The values of the various parameters for this computer run are listed in Table I.

Table I

Conventional Betatron Parameters

Run No. CONVBETA 0

Initial Beam Energy γ_o = 7.85 (3.5 MeV)

Beam Current I (KA) = 5

Major Radius r_o (cm) = 100

Initial Beam minor raduis r_b (cm) = 8

Torus minor radius a (cm) = 16

Initial beam center position r_i (cm) = 104

Betatron Magn. Field at r_o, z = o, B_{oz} (G) = 143.5

Initial emittance ε (rad - cm) = 0.400 (unnormalized)

Initial temperature spread (half width) $\frac{\Delta\gamma}{\gamma_o}$ = 0.0

External field index n = 0.447

Self field index n_s = 0.16

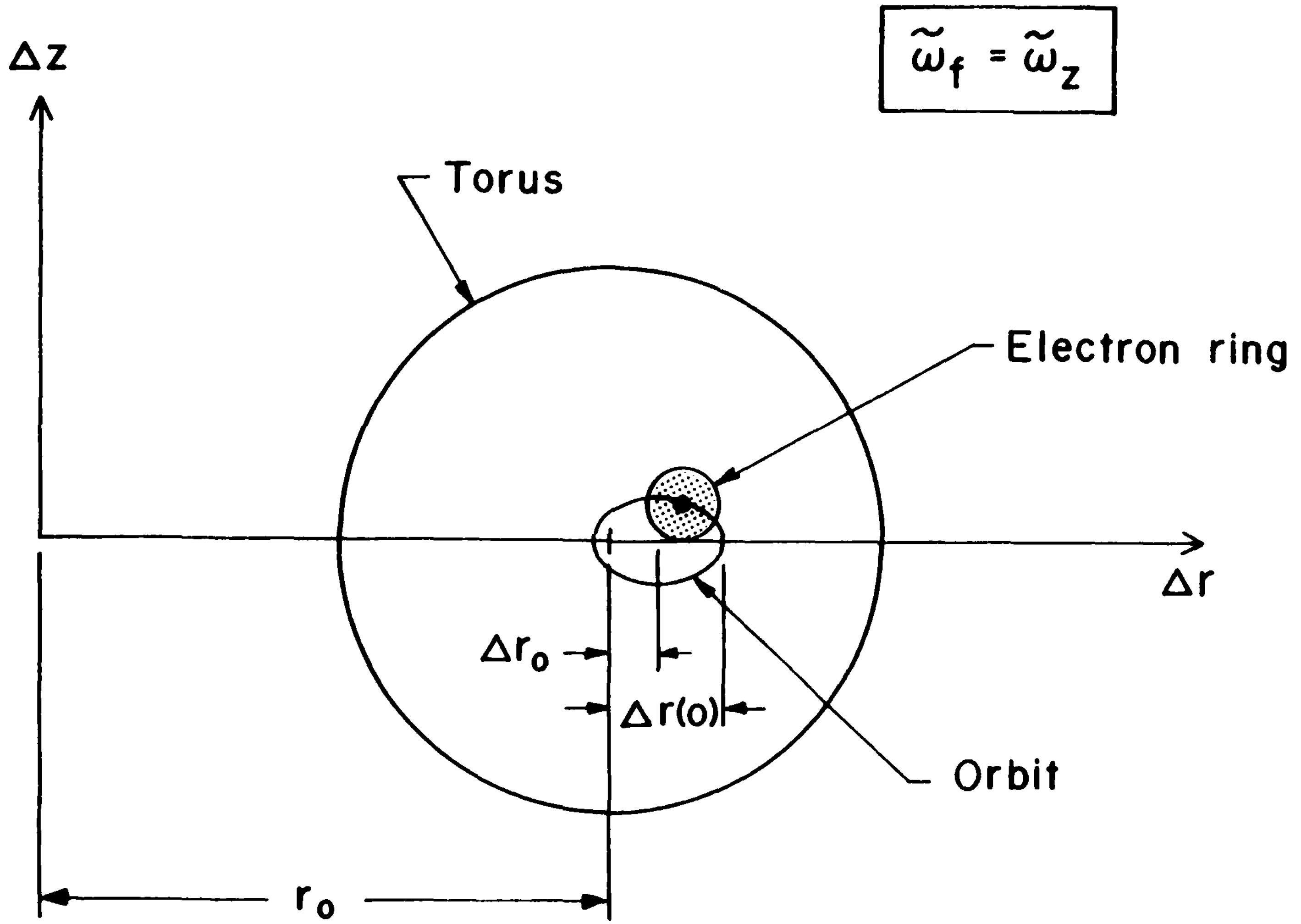


Fig. 3. Under appropriate initial conditions the center of the ring can describe an ellipse.

$$E_\rho(\Delta) = -\frac{2\,|e|\,N_\ell}{a^2}\,\frac{\Delta}{(1 - \Delta^2/a^2)}, \tag{12a}$$

and

$$B_\phi(\Delta) = -\frac{2\,|e|\,N_\ell \beta_o}{a^2}\,\frac{\Delta}{(1 - \Delta^2/a^2)} = -\frac{2\,|e|\,N_\ell \beta_o}{a}\sum_{\ell = 1}^{\infty}\left(\frac{\Delta}{a}\right)^{2\ell - 1}, \tag{12b}$$

provided that the beam does not touch the perfectly conducting wall. In Eqs. (12), $\Delta^2 = \Delta r^2 + \Delta z^2$, N_ℓ is the number of electrons per unit length in the beam and $\beta_o = V_o/c$.

Since for a completely non-neutral beam the electric field force is greater than the magnetic force, the beam density does not remain uniform whenever a section of the beam is near a conducting wall, but rather develops a peak at its outer edge facing the wall.

As a consequence of the beam density profile distortion the ratio Δ/a increases leading to larger amplitude oscillation that could result in substantial particle losses, as shown in Fig. 4. In this run at $t' = o$ the surface of the beam is more than one centimeter away from the wall and the beam center was arranged to move toward the equilibrium position. However, the wall forces reversed the direction of motion and most of the beam was lost in a short period of time. Therefore, in order to avoid the non-linearities of image forces, it is necessary to keep the electrons far away from the wall. Typically, the ratio $(\Delta + r_b)/a < 0.5$, where r_b is the beam radius.

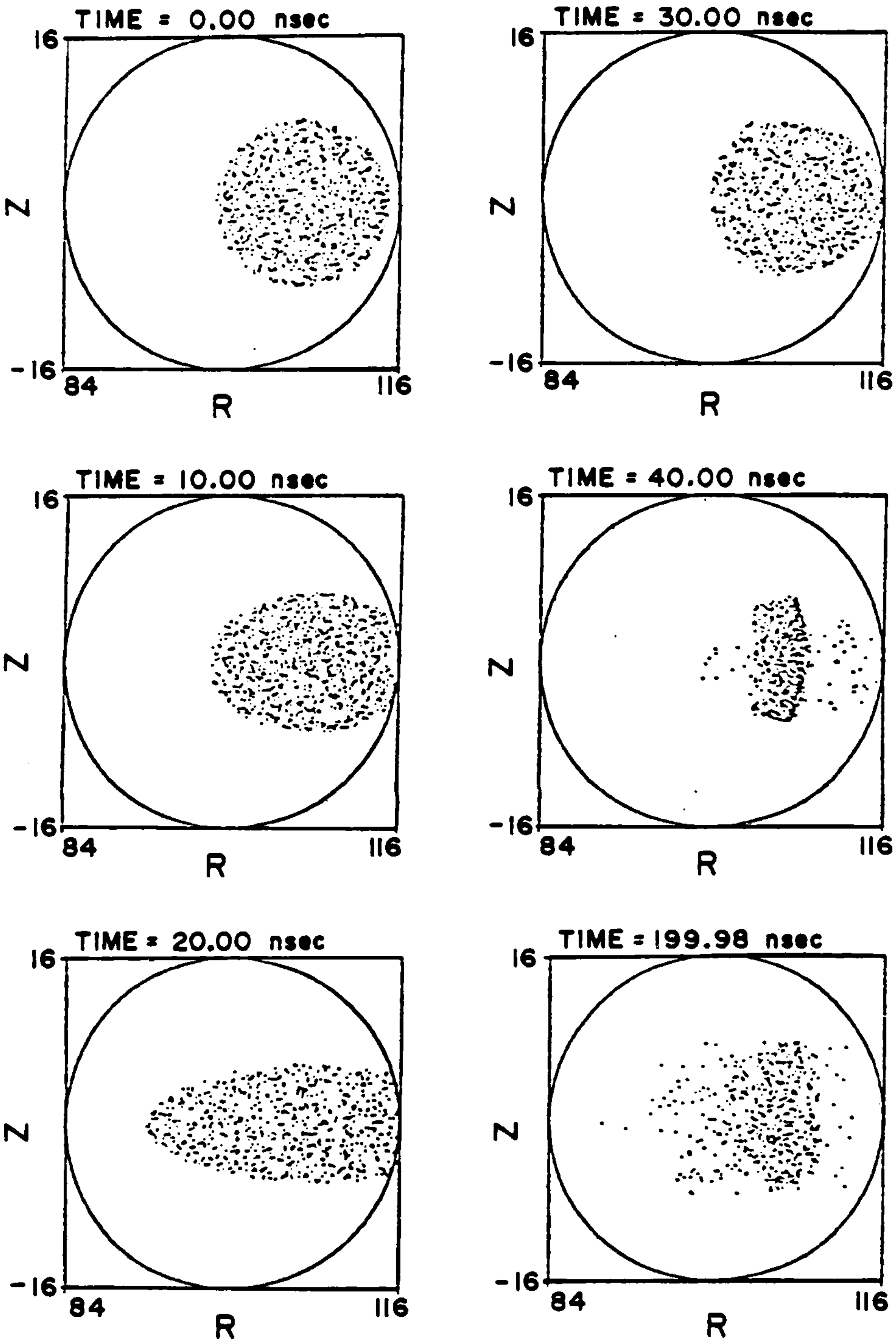


Fig. 4. Snap-shots of the electron ring minor cross-section. The non-linear image forces can have a detrimental effect on the beam when the beam surface is very near the wall. The various parameters for this computer run are listed in Table II.

Table II

Conventional Betatron

Run No. CONVBETA 08

Initial Beam Energy γ_o = 7.85 (3.5 MeV)

Beam Current I (KA) = 5

Major Radius r_o (cm) = 100

Initial Beam minor radius r_b (cm) = 8

Torus minor radius a (cm) = 16

Initial beam center position r_i (cm) = 107

Betatron Magn. Field at r_o, z = o B_{oz} (G) = 136.1

Initial emittance ε (rad - cm) = 0.200 (unnormalized)

Initial temperature spread (half width) $\frac{\Delta\gamma}{\gamma}$ = 0

External field index n = 0.447

Self field index n_s = 0.18

b. Individual Particle Motion

In the system of coordinates shown in Fig. 5, the equations describing the motion of individual electrons in a constant radius beam are

$$\delta\ddot{r} + \hat{\omega}_r^2 \, \delta r = \frac{c^2}{r_o} \, \frac{\Delta\gamma}{\gamma_o}, \tag{13a}$$

and

$$\delta\ddot{z} + \hat{\omega}_z^2 \, \delta z = o, \tag{13b}$$

where $\hat{\omega}_r^2 = (\Omega_{oz}/\gamma_o)^2 \, (1 - n - n_s)$, $\hat{\omega}_z^2 = (\Omega_{oz}/\gamma_o)^2 \, (n - n_s)$,

and $\Delta\gamma = \gamma - \langle\gamma\rangle = (V_{\theta o} \, \gamma_o^3/c^2) \, (V_\theta - \langle V_\theta\rangle)$ i.e.,

the toroidal energy spread in the beam.

For time independent fields the solution of Eqs. (13) is

$$\delta r = \delta r_o + [\delta r(o) - \delta r_o] \cos\hat{\omega}_r t + \frac{\delta\dot{r}(o)}{\hat{\omega}_r} \sin\hat{\omega}_r t, \tag{14a}$$

and

$$\delta z = \delta z(o) \cos \hat{\omega}_z t = \frac{\delta\dot{z}(o)}{\hat{\omega}_z} \sin \hat{\omega}_z t, \tag{14b}$$

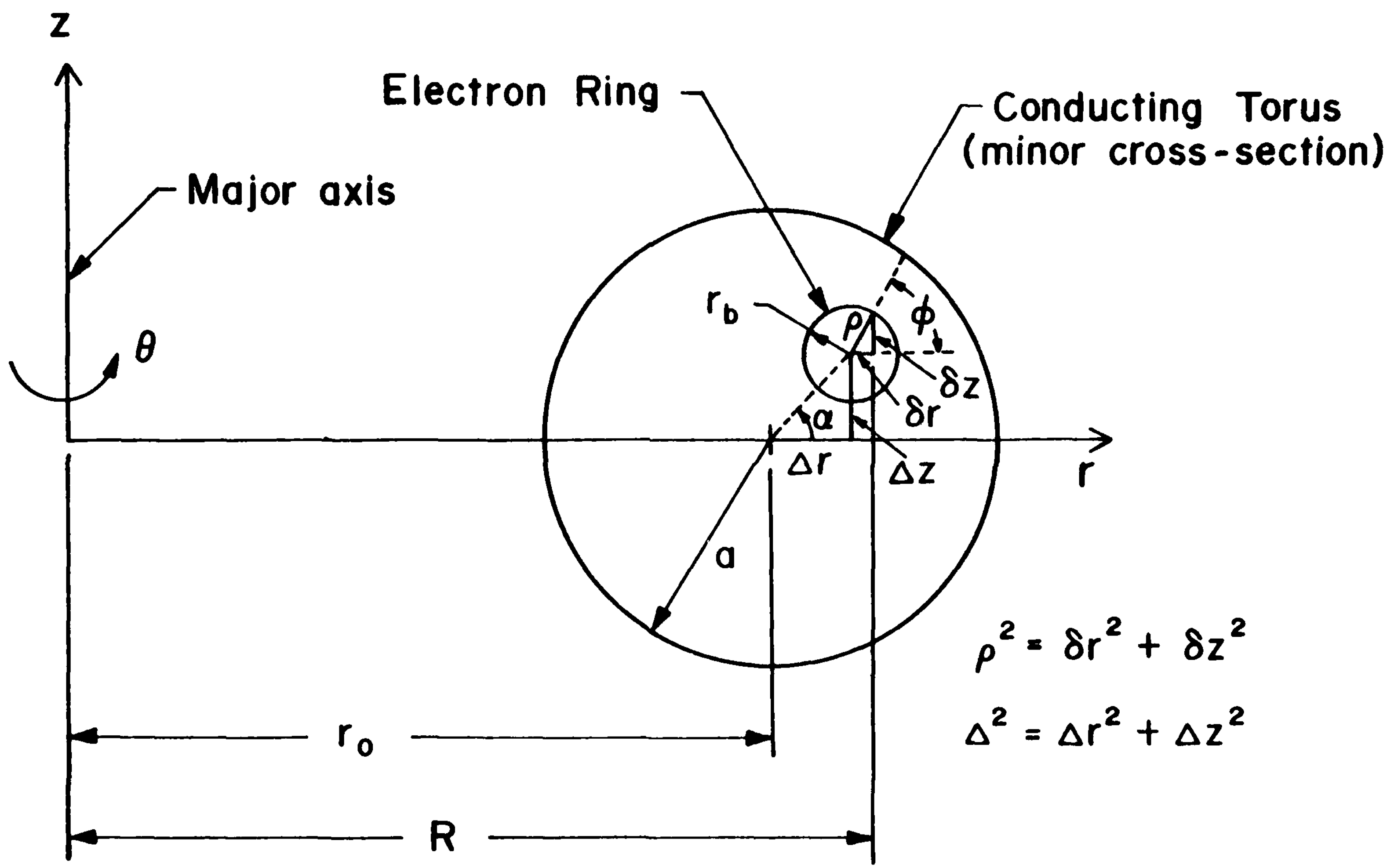


Fig. 5. System of coordinates for Eq. (13).

where $\delta r_o = (c^2/r_o\omega_r^2)\ (\Delta\gamma/\gamma_o)$ and $\delta r(o)$, $\delta z(o)$, $\delta\dot{r}(o)$, $\delta\dot{z}(o)$ are the initial position and velocity components of the particle.

For n = 1/2 the two frequencies are equal, i.e., $\hat{\omega}_r = \hat{\omega}_z = \omega$ and the rms beam emittances become

$$\hat{\varepsilon}_r^2 = \frac{16}{V_{\theta o}^2}\ \{\langle\delta r^2(t)\rangle\ \langle\delta\dot{r}^2(t)\rangle - \langle\delta r(t)\ \delta\dot{r}(t)\rangle^2\}$$

$$= \frac{16}{V_{\theta o}^2}\ \{\langle\delta r^2(o)\rangle\ \langle\delta\dot{r}^2(o)\rangle + \langle\delta r_o{}^2\rangle\ [\omega^2\langle\delta r^2(o)\rangle \sin^2\omega t$$

$$+ \langle\delta\dot{r}^2(o)\rangle\ (\cos\omega t - 1)^2]\}, \tag{15a}$$

and

$$\hat{\varepsilon}_z{}^2 = \frac{16}{V_{\theta o}^2}\ \{\langle\delta z^2(o)\rangle\ \langle\delta\dot{z}^2(o)\rangle\}. \tag{15b}$$

Equations (15) are based on the assumption that at t = o the beam is in a K - V distribution[20] and

thus $\langle\delta r(o)\rangle=\langle\delta\dot{r}(o)\rangle=\langle\delta r_o\ \delta r(o)\rangle=\langle\delta r(o)\ \delta\dot{r}(o)\rangle=\langle\delta z(o)\rangle=\langle\delta\dot{z}(o)=\langle\delta z(o)\delta\dot{z}(o)\rangle=o$.

In addition, for such a distribution, it is easy to show that

$$\langle\delta r^2(o)\rangle = \langle\delta z^2(o)\rangle = r_b{}^2/_4 \tag{16}$$

and

$$\langle \delta\dot{r}^2(o)\rangle = \langle \delta\dot{z}^2(o)\rangle = V_\perp{}^2(o)/4 \simeq \frac{\omega^2 r_b{}^2}{4}. \qquad (17)$$

Substituting Eqs. (16) and (17) into Eqs. (15), we obtain

$$\hat{\varepsilon}_r{}^2 = \varepsilon^2[1 + \frac{8\langle \delta r_o{}^2\rangle}{r_b{}^2}(1 - \cos \omega t)], \qquad (18a)$$

and

$$\hat{\varepsilon}_z{}^2 = \varepsilon^2, \qquad (18b)$$

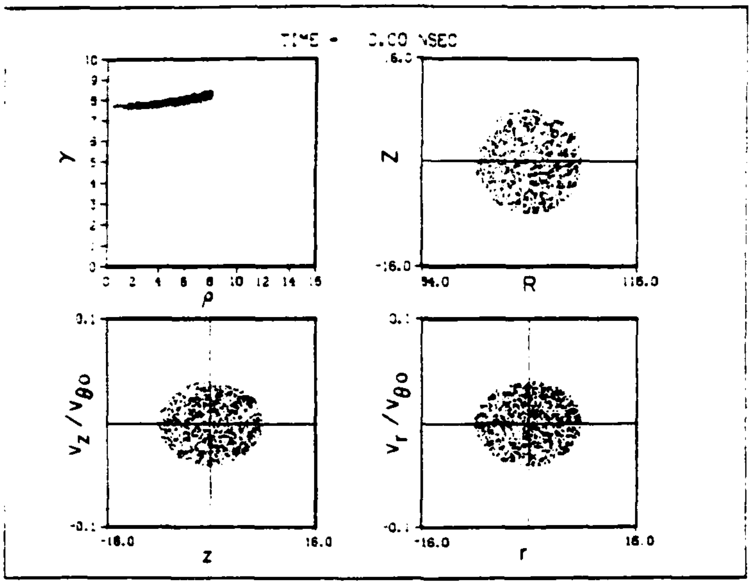
where

$$\varepsilon = r_b\, V_\perp(o)/V_{\theta o}. \qquad (19)$$

and

$$\omega^2 = (\Omega_{oz}/\gamma_o)^2\,(1 - n - n_s)\ . \qquad (20)$$

Intense electron rings with thermal energy spread have been simulated numerically. For the run shown in Fig. 6 the various parameters are listed in Table III. Figures 6a, 6b and 6c give the variation of γ with radial distance, the configuration space and the phase spaces for three different times. It is observed that the ring envelope varies sinusoidally with a peak radial amplitude that is almost twice of its initial value. This is consistent with Eq. (14a), which predicts that thermal effects will increase the radial excursions of the electrons by $2\Delta r_o$. For $r_o = 100$



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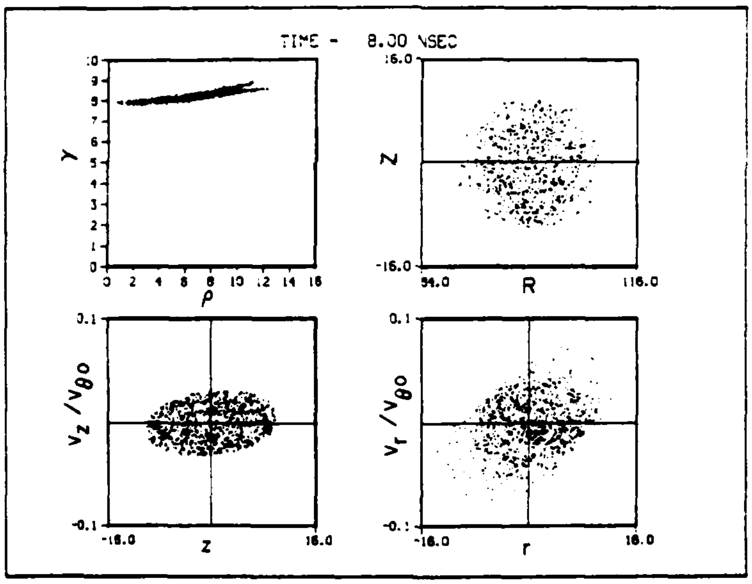
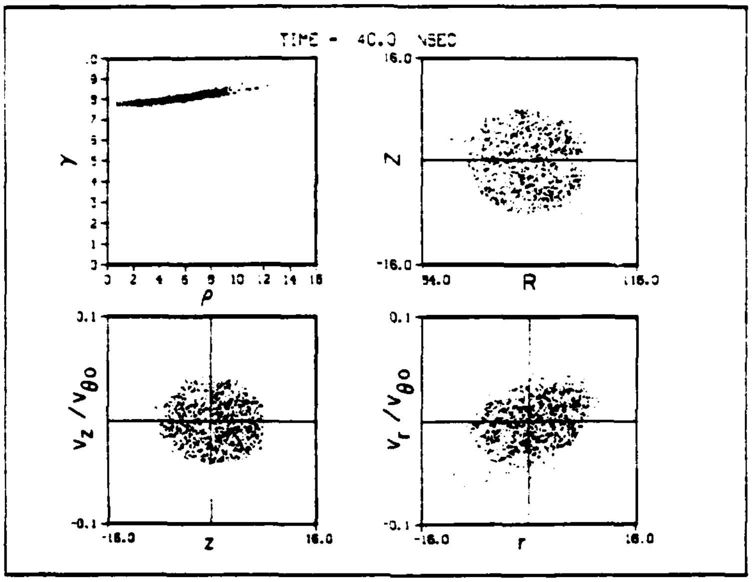


(b)

Fig. 6. Radial profile of γ, configuration space and phase space at t = o (a); t = 8 (b); and t = 40 nsec (c), for a half-width axial energy spread of 1%. The variation of the rms emittance is shown in (d). The various parameters for this run are listed in Table III.



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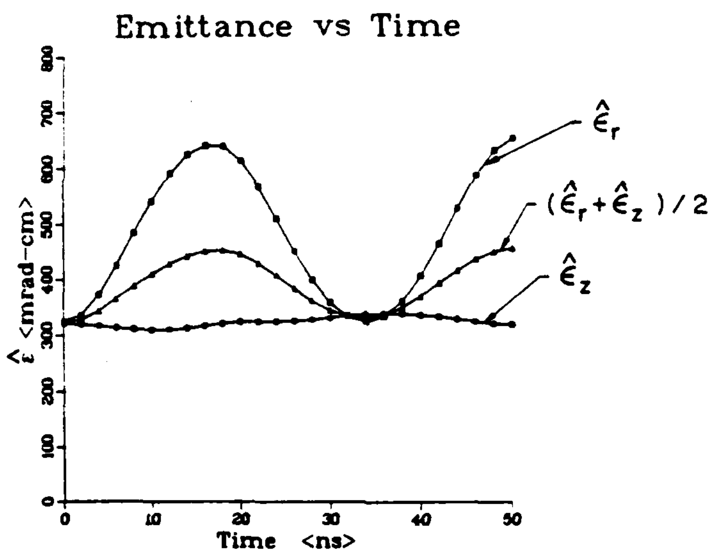


(d)

Fig. 6 (Cont'd). Radial profile of γ, configuration space and phase space at $t = o$ (a); $t = 8$ (b); and $t = 40$ nsec (c), for a half-width axial energy spread of 1%. The variation of the rms emittance is shown in (d). The various parameters for this run are listed in Table III.

Table III

Conventional Betatron

Run No. CONVBETA 04

Initial Beam Energy γ_o = 7.85 (3.5 MeV)

Beam Current I (KA) = 10

Major Radius r_o (cm) = 100

Initial Beam minor radius r_b (cm) = 8

Torus minor radius a (cm) = 16

Initial beam center position r_i (cm) = 100

Betatron Magn. Field at r_o, z = o, B_{oz} (G) = 153.7

Initial emittance ε (rad - cm) = 0.320 (unnormalized)

Initial temperature spread (half width) $\frac{\Delta\gamma}{\gamma_o}$ = 1%

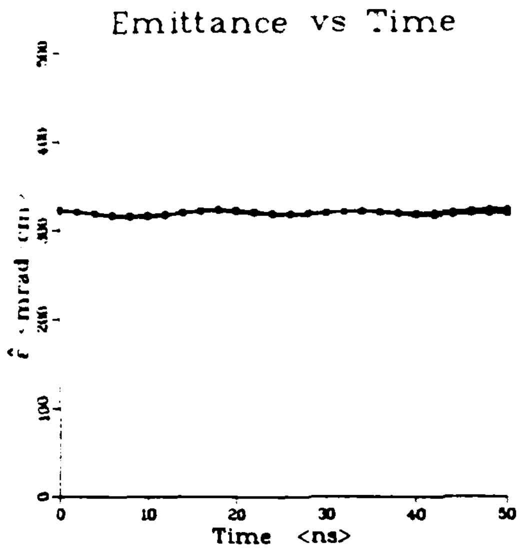
External field index n = 0.45

Self field index n_s = 0.289

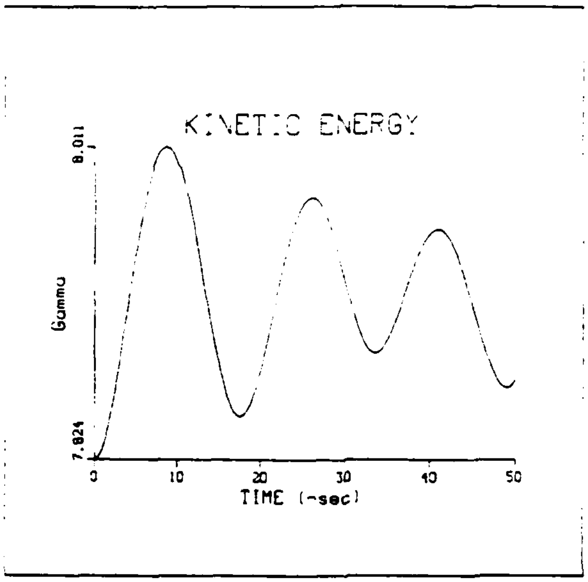
cm, $n = 0.45$, $n_s = 0.29$ and a fractional thermal half width $\delta = 1\%$, the additional radial excursion will be $2\Delta r_o = 2r_o\ \delta/(1-n-n_s) = 8$ cm, i.e., equal to the initial beam radius. In contrast, in a high current modified betatron n_s can be considerably greater than unity and thus the radial excursions can be substantially smaller.[12]

The variation of the rms emittance as a function of time is shown in Fig. 6d. In accordance with Eq. (18b), the $\hat{\varepsilon}_z$ remains approximately constant in time. However, $\hat{\varepsilon}_r$ oscillates with a period that is about 38 nsec. For the parameters of the present run Eq. (20) predicts a period of about 40 nsec. The small difference is probably related to toroidal effects. In addition to the period, the shape of the oscillations predicted by Eq. (18a) is very similar to that of Fig. 6d. Moreover, Eq. (18a) predicts a peak amplitude that is about 700 mrad-cm, which is slightly higher than the first peak of Fig. 6d.

The oscillations of $\hat{\varepsilon}_r$ are reduced practically to zero when $\delta = o$. This is shown in Fig. 7a. With the exception of parallel thermal energy spread the parameters of this run are identical to those of the previous run and are listed in Table IV. In these runs it is important to avoid to introduce an artificial energy spread as for example by using, at $t = o$, a cylindrical K-V distribution to load the electrons in the code. In such a case the electrons quickly acquire an "energy spread" during the run. This "thermalization" is due to the fact that a cylindrical K-V distribution is not suitable for high current electron rings that have large aspect ratio r_b/r_o, as may be seen as follows: For a uniform density ring that is located inside a conducting torus with its minor axis lying along the minor axis of the torus, the difference in



(a)



(b)

Fig. 7. (a) Variation of the rms emittance $\hat{\varepsilon}_r$, $\hat{\varepsilon}_z$ and $(\hat{\varepsilon}_r + \hat{\varepsilon}_z)/2$ for zero energy spread; (b) temporal variation of γ. The various parameters for this run are listed in Table IV.

Table IV

Conventional Betatron

Run No. CONVBETA 05

Initial Beam Energy γ_o = 7.85 (3.5 MeV)

Beam Current I (KA) = 10

Major Radius r_o (cm) = 100

Initial Beam minor radius r_b (cm) = 8

Torus minor radius a (cm) = 16

Initial beam center position r_i (cm) = 100

Betatron Magn. Field at r_o, z = o, B_{oz} (G) = 153.7

Initial emittance ϵ (rad - cm) = 0.320

Initial temperature spread (full width) $\frac{\Delta\gamma}{\gamma_o}$ = 0.0

External field index n = 0.45

Self field index n_s = 0.289

the potential energy between the outer and inner edge of a ring, along the midplane (z = o), is given by

$$\Delta\Phi = \frac{-|e|}{mc^2} [\Phi_{out} - \Phi_{in}] = 2\nu(r_{b}/r_{o}) \left[1/4 - \ln \frac{a}{r_b} - \frac{r_b^2}{4a^2}\right]. \tag{21}$$

For $a/r_b = 2$, $r_{b}/r_{o} = 0.008$, $\nu = 0.5$, Eq. (21) gives $\Delta\Phi = -4\%$. Of course for a cylindrical K-V distribution, $\Delta\Phi = 0$. Thus, a ring that has been incorrectly loaded, it tries to attain a more physical distribution, but in the absense of dissipation this can be achieved only temporarily. In the process a spread in γ is developed, which is equivelant to temperature.

Often, the electron ring develops transverse oscillations. These oscillations generate a toroidal electric field that modifies the kinetic energy of the gyrating electrons according to the equation

$$mc^2 \frac{d\gamma}{dt} = - |e| \vec{V}.\vec{E} \simeq |e| (V_{\theta}/c) \left(\frac{dA_\theta}{dt}\right). \tag{22}$$

The change in γ can be obtained by integrating Eq. (22). Assuming that the beam is located at the center of the torus, A_θ is given

$$A_\theta = (2I/c) [1/2 + \ln (a/r_b)]. \tag{23}$$

For $\gamma^2 >> 1$ the result is

$$\gamma_f - \gamma_{in} = 2\nu \ln \left(\frac{r_{bf}}{r_{bin}}\right), \tag{24}$$

where γ_f, γ_{in} are the final, initial values of γ and r_{bf}, r_{bin} are the final, initial values of the ring radius. The variation of γ as a function of time for the run of Table IV is shown in Fig. 7b. Equation (24) predicts a $\gamma_f - \gamma_{in} \simeq 0.2$, which is in good agreement with the numerical results.

Combining Eqs. (17), (19) and (20), and assuming that $n_s << 1/2$ and $n = 1/2$, we obtain the maximum emittance that is allowed in a betatron of major radius r_o and is

$$\varepsilon_b^2 < 1/2 \left(\frac{r_b^4}{r_o^2}\right).$$

The maximum emittance that can be accomodated in a emittance dominated beam confined in a modified betatron is considerably greater and is given by

$$\varepsilon_{mb}^2 < (r_b^4/r_o^2)\ (B_{o\theta}/2B_{oz})^2.$$

The ratio of the two emittances is $\frac{\varepsilon_{mb}}{\varepsilon_b} = \frac{B_{o\theta}}{r_2 B_{oz}}$ and in general it is much greater than unity.

c. Toroidal Corrections[8,12,14]

The cause of these effects is the finite curvature of the electron beam orbit. For relatively large aspect ratio $r_o/r_b >> 1$ beams, the toroidal effects become important when ν/γ_o exceeds a few percent. The toroidal corrections have been discussed extensively in relation to the Modified Betatron. The fields at the center of a uniform charge and current density

electron ring inside a perfectly conducting toroidal chamber of circular cross-section are

$$\underset{\sim}{E}_{ind} = -2\pi|e|n_o r_o [(\frac{r_b^2}{a^2}\frac{\Delta r}{r_o} + \frac{1}{2}\frac{r_b^2}{r_o^2}\ln\frac{a}{r_b})\hat{e}_r + \frac{r_b^2}{a^2}\frac{\Delta z}{r_o}\hat{e}_z], \tag{25}$$

and

$$B_{ind} = -2\pi|e|n_o\beta_o r_o [\frac{r_b^2}{a^2}\frac{\Delta z}{r_o}\hat{e}_r - (\frac{r_b^2}{a^2}\frac{\Delta r}{r_o} - \frac{r_b^2}{2r_o^2}(1 + \ln\frac{a}{r_b}))\hat{e}_z], \tag{26}$$

where n_o is the ambient density, $\beta_o = v_o/c$, v_o is the azimuthal velocity defined by

$$v_o = \frac{r_o\Omega_{oz}/\gamma_o}{1 + 2(\nu/\gamma_o)(1/2 + \ln a/r_b)}, \tag{27}$$

and the displacement $\Delta r, \Delta z$ of the ring from the center of the torus has been assumed to be much less than a.

Using the fields of Eqs. (21) and (22), it can be shown that the center of the beam is described by the equations

$$\ddot{\Delta r} + \omega_r^2\,\Delta r = \frac{c^2}{r_o}\frac{\delta\gamma_o}{\gamma_o}, \tag{28}$$

and

$$\ddot{\Delta z} + \omega_z^2\,\Delta z = o, \tag{29}$$

where

$$\omega_r^2 = (\Omega_{oz}/\gamma_o)^2 \, [\xi^2 - n\xi - 2\nu \, c^2/\gamma_o^3 a^2 (\Omega_{oz}/\gamma_o)^2], \tag{30}$$

$$\omega_z^2 = (\Omega_{oz}/\gamma_o)^2 \, [n\xi - 2\nu c^2/\gamma_o^3 a^2 \, (\Omega_{oz}/\gamma_o)^2], \tag{31}$$

and

$$\xi = \{1 + (2\nu/\gamma_o) \, [0.5 + \ell n \, (a/r_b)]\}^{-1}.$$

According to Eqs. (24) and (26), the equilibrium position of the orbit is displaced from the center of the minor cross-section of the torus, whenever the energy mismatch $\delta\gamma_o$ is not zero. The displacement is

$$\frac{\Delta r_o}{r_o} = \frac{c^2(\delta\gamma_o/\gamma_o)}{r_o^2 \, \omega_r^2} = \frac{\gamma_o \delta\gamma_o / \, (\gamma_o^2 - 1)}{[1 - n/\xi - 2\nu \, r_o^2/\gamma_o a^2 (\gamma_o^2 - 1)]}.$$

The above equation predicts that
for $\delta\gamma_o/\gamma_o = 1\%$, $\gamma_o = 5$, $r_o/a = 7$, $n = 1/2$, $\nu/\gamma_o = 0.059$, i.e., for $I = 5$ KA, the ratio $\Delta r_o/r_o = 0.05$, which for $r_o = 110$ cm gives a displacement $\Delta r_o = 5.5$ cm.

An interesting manifestation of toroidal effects is in the value of betatron magnetic field required to confine the rotating beam at a specific radius. When the axis of the beam lies along the axis of the torus, i.e.,

when $\Delta r = \Delta z = o$, it can be shown from Eqs. (25) and (26) that the external magnetic field required for the beam to rotate with a radius r_o is

$$B_{oz} = B_o \left\{1 + 2\nu/\gamma_o \,(0.5 + \ell n \; a/r_b)\right\}, \qquad (32)$$

where B_o is the magnetic field necessary for a single particle of the same kinetic energy to rotate with a radius r_o.

For the run of Table IV the single particle magnetic field is 134G, about 20G lower than that used in the simulation. Equation (32) predicts that the required field B_{oz} is 157.8G, approximately 4G higher than that of Table IV. The difference is related to the fact that Eq. (32) was derived under the assumption that the ratio $r_{b/a} \ll 1$, which is not satisfied.

III. Self Magnetic Field Diffusion

To allow the external accelerating magnetic field to penetrate inside the torus, the vacuum chamber is constructed from materials with finite conductivity. As a result, the self magnetic field diffuses out the chamber for times comparable with the magnetic diffusion time t_D. The inductive electric field generated by the changing flux acts to slow down the beam. In addition, the hoop forces increase and the induced magnetic field components (image fields) go to zero at the end of the diffusion. However, the induced electric field components (image fields) remain the same. Although these two effects change the equilibrium position of the beam in the opposite direction, in general they do not balance each other and thus the equilibrium can be lost. This difficulty can be avoiding by compensating for the diffusion of the field with external circuits.

However, it is very unlikely that the compensation can be perfect. Therefore, in practical situations it is desirable to know the maximum permissible error in the compensation that will not result in the loss of the equilibrium.

Consider a electron ring, which for $t \ll t_D$ is inside a finite conductivity toroidal chamber with its minor axis lying along the minor axis of torus, as shown in Fig. 8. Initially, the magnetic boundary coincides with the electric boundary and has a radius equal to a. As a result of the incomplete compensation the magnetic boundary moves, at $t \sim t_D$ to a new radius a', but the electric boundary remains at its initial position. The reduction in γ associated with the shift of the magnetic boundary can be computed from

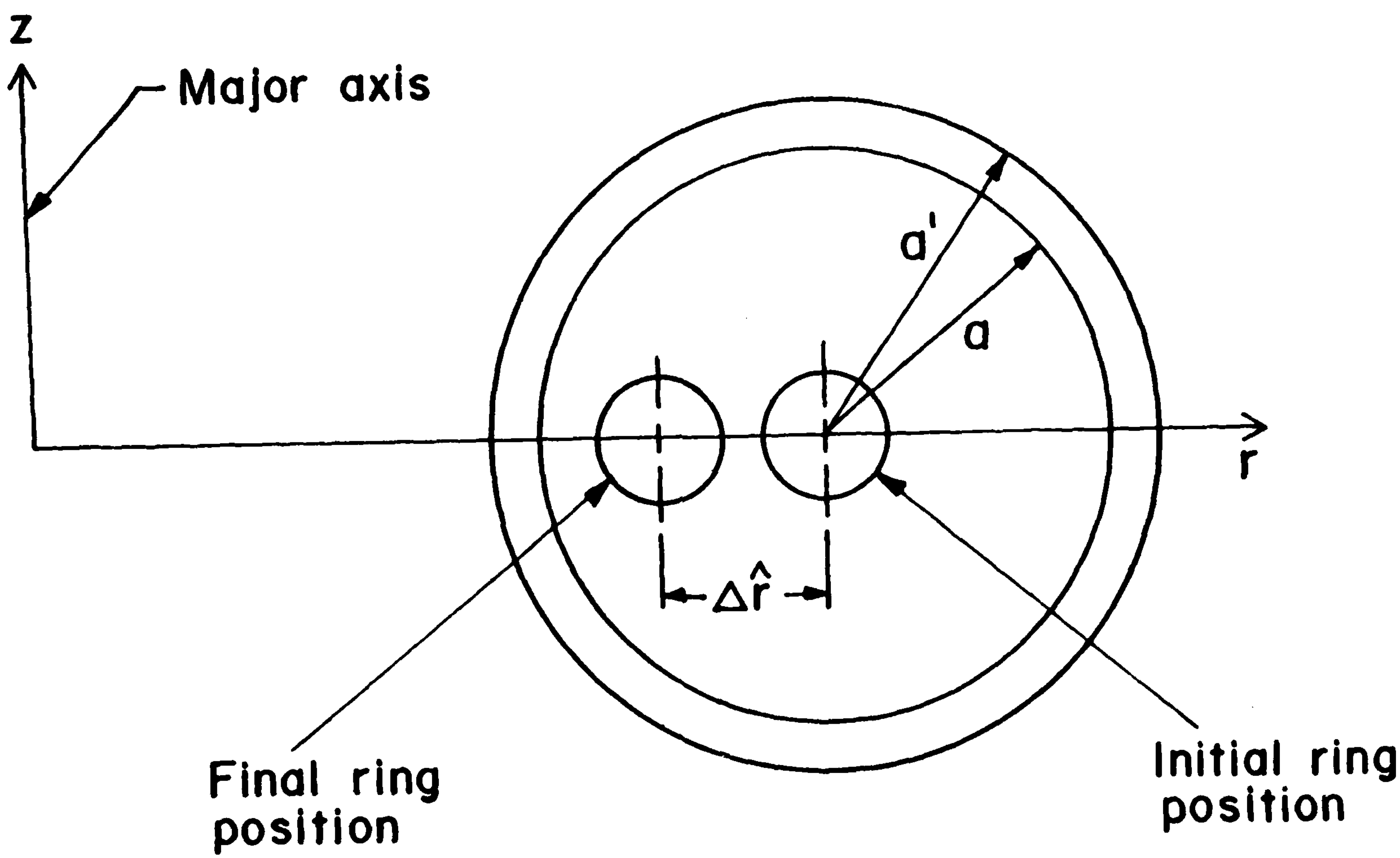


Fig. 8. Initial and final ring position resulting from the incomplete compensation of wall currents during diffusion of self magnetic field.

Eq. (22) and is given by

$$\Delta\gamma = -2\nu\, \beta_\theta^{\,2}\, \ell n\, (\frac{a'}{a}) \ . \tag{33}$$

In addition to the reduction of γ, the hoop forces will increase and therefore the equilibrium position of the ring will move a distance $\Delta\hat{r}$ from the center of the torus, which was the initial equilibrium position. This distance can be computed from the radial balance equation using the fields of Eqs. (25) and (26) and the reduction of γ given by Eq. (33). The result is

$$\frac{\Delta\hat{r}}{r_o} = \frac{\nu/\gamma\ \ell n\ (a'/a)}{\left[1-n-\ (n_s r_b^{\,2}/a^2)\ (1+\ 2\gamma^2\ \delta a/a)\right]},$$

where $\delta a = a'-a$.

In order to keep the displacement $\Delta\hat{r} \ll a$ it is necessary to avoid the singularity of the denominator,i.e.,

$$(n_s r_b^{\,2}/a^2)\ (1 + 2\gamma^2\ \delta a/a) \ll 1-n.$$

For $\delta a/a = 10\%$, $\gamma = 7$, $n = n_s = 1/2$, the above relation gives

$$(\frac{r_b}{a})^2 \ll \frac{1}{11} \ .$$

The corresponding displacement of the equilibrium position for $a = 15$ cm, $r_o = 100$ cm and $\nu = 0.59$ is 1.6 cm. Therefore, the shift in the equilibrium position during diffusion is manageable, provided that the compensation is better than 90%.

IV Conclusions

The main conclusions that may be drawn from the present studies are: Like the modified betatron, the conventional betatron is sensitive to the energy mismatch and the diffusion of the self magnetic field. However, in contrast to the modified betatron, the conventional betatron cannot accomodate large thermal energy spread and large emittance. These advantages of the modified betatron, together with its superior stability[21-23] properties make it a more appropriate accelerator when intense beams are desired.

Finally, it is necessary to keep the ratio $r_{b/a} \ll 1$, in both devices, in order to avoid very unpleasant surprises, in particular in the high current regime.

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Appendix A

Consider a straight electron beam of radius r_b inside a cylindrical, conducting pipe of radius a as shown in Fig. 5. The total vector potential $\vec{A}_T = \vec{A}_p + \vec{A}_h$, where $\vec{A}_p$ is the particular and $\vec{A}_h$ is the homogeneous solution of the wave equation.

When the displacement current is neglected, the particular solution for $\rho > r_b$ is

$$\vec{A}_p\ (\rho,\ \phi,\ t) = -\frac{2I(t)}{c}\ \ell n\left|\vec{\rho} - \vec{\Delta}\ (t)\right|\hat{e}_\theta, \tag{A-1}$$

where

$$\left|\vec{\rho} - \vec{\Delta}(t)\right| = \left[\rho^2 + \Delta^2(t) - 2\rho\Delta(t)\ \cos\ (\phi - \alpha(t))\right]^{1/2}$$

and $I(t) = -\left|e\right|\ n_o\ v_o\ \pi r_b^{\ 2}$ is the beam current.

Similarly, the homogeneous solution is

$$\vec{A}_h\ (\rho,\ \theta,\ t) = \sum_{\ell = o}^{\infty} a_\ell(t)\cdot(\rho/a)^\ell e^{i\ell\phi}\ \hat{e}_\theta + \text{c.c.}, \tag{A-2}$$

where the coefficients are to be determined from the boundary conditions. For a perfect conductor $A_T = o$ at $\rho = a$ and Eqs. (A-1) and (A-2) give

$$a_o = (I/c)\ell na$$

$$a_\ell = -(I/c)\,\ell^{-1}\,(\Delta/a)^\ell\,e^{-i\ell\alpha},\quad \ell = 1,2\cdots$$

The magnetic field at the center of the beam is

$$B_\phi(\Delta,\alpha) = -\frac{\partial A_h}{\partial\rho} = -\frac{2I}{ac}\sum_{\ell=1}^{\infty}\left(\frac{\Delta}{a}\right)^{2\ell-1}$$

$$= -\frac{2I}{\alpha^2 c}\,\frac{\Delta}{(1-\Delta^2/a^2)}\,. \tag{A-3}$$

Equation (A-3) was derived without any assumption about the beam radius.

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